INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH IN MULTIDISCIPLINARY EDUCATION
ISSN(print): 2833-4515, ISSN(online): 2833-4531
Volume 02 Issue 08 August 2023
DOI: 10.58806/ijirme.2023.v2i8n08
Page No. 356-367

Application of Lotka-Volterra's Predator – Prey Model in the Solution of Herdsmen – Farmers Crisis in Nigeria

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ABSTRACT: Social Scientists and Statisticians can possibly investigate the causes and proffer solutions to population phenomena such as the crisis between farmers and Herders in Nigeria by use of statistical data and modern statistical tools, but looking at the same problem mathematically becomes very difficult. To this end logistic models can be formulated to handle the problem mathematically. Therefore, this research focuses on solving the problem by developing a population model that assigns parameters to the species involved and formulating a model reducible to differential equations. This was done by employing Lotka-Volterra's predator – prey model to determine the equilibrium point at which both the farms and Herders will operate in peace.

INTRODUCTION

This research focuses on how to solve the problem of Fulani herdsmen and the local farmers in Nigeria, a situation that have recently developed into a worrisome abattoirs. Mathematical researches in this area have not sufficiently attracted the attention of scholars. Solution to this problem can be proffer through the use of population model by assigning parameters to represent the species involved in the problems. Suppose we are looking at the growth of a given population, we let $p(t)$ denote the population of the given species at time t and $r(t, p)$ denote the difference between its birth and death rate. If there is no net immigration and emigration then $\frac{dp}{dt}$ (the rate of change) of the population will be $rp(t)$. In a simplest model we assume that r is constant, that is, it does not change with either time or population. In such a case, the following differential equation becomes the governing model for the population growth.

$$\frac{dp(t)}{dt} = ap(t) \quad (\text{a = constant}) \qquad (1)$$

is a linear equation known as the Malthusian law of population growth.

Most ordinary differential equations are generated in the context of geometry, mechanics, astronomy or population. Predator-prey models are developed by focusing primarily on population variables and on the assumptions that other less impacting variables do not exist.

Predation models focus on factors such as the 'natural' growth rate, or birth rate, and the carrying capacity of the environment in which the population resides. Here under consideration, will be majorly population decreasing variables such as predation, (killing rate).

The idea is to draw attention to the lingering problem of nomadic migration in Nigeria. Think about the daily life of the human race. In most time, effort and energy are put forth toward making money in some ways, shape or form. A process which involves being educated, working as farmers, trading or networking. Majority of human activity is spent on making money, which equivocates to the survival of the individual. This major concern on survival is what drives theories of natural selection, evolution, and co – evolution.

This research focuses on applying mathematics to analyzing predation relationships, especially between the cattle Fulani herdsmen and the local farmers. This predation relationship is quite special, because these species interact in a relatively isolated manner compared to others, meaning that their populations fluctuate in a regular cycle due to lack of significant external variables on the relationship. This population fluctuation can be defined and analyzed mathematically using systems of linear ordinary differential equations built upon several minimizing assumptions in order to exclude incalculable variables. To achieve this set gaol we will employ a Mathematical model called the Lotka-Volterra predator – prey model, which can solve real life problems either analytically or by using computer simulation to determine the dynamic nature of the population through periodic lengths, phase portraits, critical points analysis and giving other practical information relevant to the reality of the relationship. **This research is concerned with analytic formulation which may not necessarily proffer numerical solution.**

The ability to analyze and predict such relationships mathematically can be quite useful to humanity in the study of extinction through predation, or even excessive co - evolution based on population interaction.

In recent times the issue of violent clashes and instability between farmers and nomadic herdsmen across many regions of Nigeria have become a major concern to communities, Governments (international and national) and indigenous developmental organizations. This, to a large extent may affect the achievement of sustainable development which aims at ending hunger, achieving food security, improving nutrition and promoting sustainable agriculture by year 2020 as proposed by the Federal government of Nigeria. The clashes between Fulani herdsmen and the local farmers in Nigeria, instead of abating have been on astronomical increase to the dismay of helpless Nigerians. Nigeria has witnessed series of violent communal clashes arising from the activities of the nomadic herdsmen who move about on a daily basis with their cattle in search of water and green pastures across boundaries. They graze on the streets and school compounds in most of the cities, and could also be found operating in remote villages in nearly all the states of the country. These nomads who are essentially Fulani tribesmen were originally found in small make-shift communities scattered across the northern fringes of Nigeria and other countries in West Africa.

Eniola, (2007), put it undisputedly, that the Fulani herdsmen represent a significant component of the Nigerian economy. They constitute the major breeders of cattle; the main source of meat, the cheapest source of animal protein consumed by Nigerians. They dominate over 90% of the nation's livestock population which accounts for one-third of agricultural GDP and 3.2% of the nation's GDP.

Thus, we can say that the contribution of the Fulani to the local food chain and national food security cannot be overstressed. The types of conflicts for survival between the Fulani herdsmen and farmers in Nigeria vary in form and intensity from one community to another. Social and economic factors continue to provoke violent conflicts among the Fulani pastoralist and farmers. The intensity and variation of the conflicts largely depend on the nature and type of the users ground where the herdsmen graze. These conflicts have continued to constitute serious threats to the means of survival and livelihood of both the farmers and herdsmen which both groups are tenaciously protecting and projecting.

It has been established from community reports around in parts of the country, including Edo states, that besides the destruction of crops by the cattle herdsmen, they have been found to be involved in crimes like kidnapping, armed robbery, murder, rape, bush burning and stealing. This act has resulted in clashes between them and the host communities. On the other hand, pastoralists have lost their lives and property in an orgy of killings and destruction that is not only affecting livelihood but also affecting national cohesion. Several local government areas have been listed where the attacks have been consistent; examples in Edo State include; Ugboha, Esan South-East local government area, Odighi and Odiguetue in Ovia North-East local government area; and Auchi in Estakor local Government area. Some researchers have related the causes of conflict to the global climate change and the contending desertification and aridity nature of the area that have reduced arable and grazing lands, forcing pastoralist to move southwards in search of pastures for their livestock. The work of Okoli, Enyinonla and Elijah, (2014); Odoh and Chigozie (2012); Abass, (2012). And that of Nformi *et al*, (2014) revealed that farmers' encroachment on cattle routes is one real cause of clashes between them and nomads.

Some scholars also have identified different factors which include; climate changes, the migration further south, drastic reduction in the growth of agro-pastorals, the expansion of farming on pasture-rich lands, the invasion of farmlands by cattle, assault on non-Fulani women by herders, blockage of livestock routes and water points, fresh water scarcity, ethnic stereotyping, and the breakdown of conflict intervention mechanisms as the root causes of such violence in rural areas as found in the work of (Fulani 2009; Ofuoku and Isife 2009; Adekunle&Adisa 2010; Blench 2010; Odoh&Chigozie 2012; Solagbeni 2012; Audu 2013; 2014; Bello 2013; Mc Gregor 2014) explained the situation in detail, the level of Herdsmen activities in Nigeria.

Despite, the spate of violent clashes between nomadic herdsmen and farmers in Nigeria, adequate mathematical research works have not been conducted to find out the implication of these clashes on the host communities, particularly with respect to impact on population. It is against this background that this study attempts to examine the demographic implication of nomadic herdsmen and farmers clashes in Nigeria by provide a mathematical model that will examine the system of interaction and the underlying causes of the conflict between farmers and herders using Lotka-Volterra predator- prey model developed for solving population logistics.

The origin of Lotka-Volterra Model

According to Lucas and Pulley(2011), in the 1920's, a biologist named Humberto D'Ancona was doing a statistical study on the numbers of each species sold at three main Italian ports. During his study of these fish species from 1914-1923, he came across a startling revelation, He believed that these predator - prey relationships between the sharks, rays, and fish were in their natural states outside of human interaction, before and after the war. D'Ancona asked his father in-law, Vito Volterra, a very successful mathematician, to anazlyze the data and draw some conclusions. Volterra spent weeks developing a series of models for interactions of two or more species. Almost at the same time, Alfred J. Lotka, an American mathematical biologist discovered and developed the same conclusions and models as Volterra.

In their formulation, they adopt the variables x and y to denote the populations of the prey and predator, respectively and t for time. The formulation of the model was based on the following assumptions:

that in the absence of the predator, the prey grows at a rate proportional to the current population; thus:

$$\frac{dx}{dt} = ax, a > 0, \text{when } y = 0 \qquad (2)$$

that in the absence of the prey, the predator dies out, thus: $\frac{dy}{dt} = -cy, \mathrm{c} > 0 \text{ when } x = \mathrm{o}$ (3)

that when predators and prey interact, their encounters are proportional to the product of their populations, and each encounter tends to promote the growth of the predator and inhibit the growth of the prey. Therefore, the predator population will increase by zxy and the prey population will decrease by -zxy.

As a consequence of these assumptions, the Lotka-Volterra model was created, thus:

$$\frac{dx}{dt} = ax - bxy = x(\mathrm{a} - \mathrm{by}) \qquad (4)$$

$$\frac{dy}{dt} = -cy + pxy = y(-\mathrm{c} + px) \qquad (5)$$

The constants a, b, c, and z are all positive, where a and c are the growth rate constants and b and p are measures of the effect of their interactions.

In order to find trajectories of the system, the variable t is eliminated; giving rise to the following equation:

$$\frac{dy}{dx} = \frac{y(-c + px)}{x(\mathrm{a} - \mathrm{by})} \qquad (6)$$

On separating the variables and integrating, an equation defining the trajectories is formed, and if the constants are changed and varied, different trajectories will appear.

In our investigation, we discovered that the Fulani and farmers co-existed in their natural state living in peace before the Buhari administration in Nigeria with little or no cohesion. Which means this model can be adopted to proffer solution to the crisis between the Fulani and the farms putting the time frame into consideration.

Come to think of it, Pastoralists are people who live mostly in dry remote areas. Their livelihood depends on their livestock. According to them, pastoral systems take many forms and are adapted in a particular natural, political and economic environment. The key feature qualifying pastorals is mobility. The term nomadic is used when mobility is high and in an irregular pattern. (Antonio and Silvia, 2009).

According to Blench (2010), the Fulbe (another name for Fulani) and the arable farmers among whom they move, have an interdependent relationship based on the exchange of diary product for grain, and a market for the animals that must be periodically sold to provide cash for domestic purposes. Although the goods or services the pastoralists have to offer to the farming community are essential, the pastoralists are obliged to remain on good terms with farmers if they wish to continue to exploit the same locale in successive years. If herders are unable to build up and exchange relations with farming communities, then their survival will become sedentary, by their flexible movement patterns that involve exploiting new arable communities every year, they cannot but intimidate the farmers to get the best for their cattle. The savannah zone of Nigeria has abundance of forages, and the absence of the deadly cattle disease called trypanosomiasis makes the zone conducive for rearing cattle (Ibrahim et al, 2004).

Seventy five percent (75%) of today's food in Nigeria comes from 12 arable crops and 5 are animal species, with just 3 arable crops (rice, maize and wheat) accounting for about 60 percent of the calories and protein obtained from plants (Lambrou and Laub, 2006). Production of crops has been essentially the prominent features of agricultural activities. Indeed, almost all farmers in Nigeria cultivate one or more arable crops for food and income. According to Fayinka (2004), the average farm size in Northern Nigeria for arable crop production is inadequate Therefore; the scramble for this land has become a major source of conflict in Nigeria today.

THE CONCEPT OF CONFLICT

The concept of conflict is not self- explanatory, there are two ways to understand conflict: (i) an opposition of position which does not include violent action, (ii) an incompatibility of positions involving violent behaviour or hostile attitudes (Swanstrom and Weissmann 2005; Zartman 1991).

According to (Giesen 1993), the opposition between two or more individuals or groups with opposed interests concerning the way to deal with an issue might lead to violence conflict. The intensity of a conflict can vary over time and follow very different evolution patterns.

(Nohlen and Schuitze 2005a), opined that, the intensification of a conflict resulting in the use of violence, is usually called escalation, while de-escalation happens when actors turn to using less violent means of opposition or stop using violence.

Conflict is not simply a disruption of harmony but also has the useful social function of calling attention to social taboos and tensions, opening communication, and bringing forward the transformation of a society, for instance in the distribution of power (Schönegg and Martel 2006). Conflict in general belong to all societies and can even be considered as a motor for societal change and progress (Giesen 1993). Scientific movement started with a focus on natural processes and was broadened to social processes in the late nineteenth and early twentieth century as a result of conflict. Darwin explained conflict as evolutionary processes by the "survival of the fittest" in a hostile and competitive environment. In the social sphere, a context of competition over values or resources and the resulting conflict between social groups can be the trigger for innovation, adaptation, and the creation of new societal organizations (Giesen 1993). **It is however important to avoid determinism and to remember that a changing society does not necessarily tend towards progress and that evolution toward less civilized forms of conflicts are possible** (Giesen 1993).

Conflict can either bring about constructive problem-solving or escalate into violence. It was stated that farmers-herders conflicts oppose groups of population who are differentiated by their livelihoods and by the activities from which they derive their subsistence. Herders or pastoralists depend largely on mobile and extensive livestock herding, while farmers in Africa are mostly sedentary and live on small plots and from rain-fed agriculture (Bauer 2011; Lisk 2009). This proximity creates interactions and sometimes conflicts because of resource allocation. Conflict might arise between farmers and herders, or between herders, between farmers, or between the central administration and both groups (Ejigu 2009: 890). The focus here is on the conflicts between farmers and herders that are caused by resource allocation and scarcity.

Here, our primary study is on human and herbivores species which relied heavily on plant as a feeding source. Therefore variables such as crops and cattle will be regarded as parameters in developing our model.

DEVELOPMENT OF THE PREDATOR-PREY MODEL

This study involves the investigation of the cyclical relationship among the cattle herders, cattle, farmers and crops using a predator-prey interactive system of equation. Predator-prey models are the building blocks of the bio and ecosystem as biomasses are grown out of their resource masses. Species compete, evolve and disperse for the purpose of seeking resources to sustain their struggle for their very existence. They deal with the general loss-win interaction and hence may have application outside of ecosystems. This model is traced from its origin in the Malthus-Verhulst logistic equations, through the Lotka-Volterra equations with logistic modifications to both prey and predator equations taken from the Michaelis-Menten-Holling functional responses and per-capita rate of change functions. Verhulst equation was published after Verhulst had read Thomas Malthus' essay on the principle of population. Verhulst derived his logistic equation to describe the self-limiting growth of a biological population. Many populations in nature follow a logistic growth (pointed out by Thomas Malthus in the exponential growth model). The lotka-volterra model of predator-prey interactions was developed by lotka (1925) and volterra (1926). Vito Volterra (Italian mathematician) proposed a differential equation model to explain the observed increase in predator fish (and corresponding decrease in prey fish). He was inspired through his interaction with the marine biologist Umberto D'Ancona, who studied the fish catches in the Adriatic Sea and had noticed that the predatory fish caught, had increased during the years of the world war 1 (1914-1918). At the same time in the United States, the equations studied by volterra were derived independently by Alfred J Lotka to describe a hypothetical chemical reaction in which the chemical concentrations oscillate, he extended the model, through Andrey Kolmogorov, to organic systems, using a plant species and a herbivorous animal species as an example, after which he utilized the equation to analyze predator- prey interactions in his book on biomathematics. This model is based on linear per capita growth rates. It is a non-linear system of differential equations, thus the model was the first ever mathematical model on the population which achieved a balance in a population in a very cyclical way. Therefore, it is sufficient to introduce this model in the mathematical approach to represent the different types of mechanism between the predator and its prey by taking a case study of cattle herders, cattle, farmers, and crops.

This study will formulate a mathematical model that will analyze the conflict between farmers and nomadic herdsmen in Edo state and construct a model that will:

(a) find out the root causes of the conflict;

(b) analyze the Sequence of interaction and conflict between herdsmen and farmers resulting in crisis within the area of interaction and

(c) give a numerical interpretation to the problem using appropriate mathematical software that will reveal the impart of conflicts on the communities so attacked and the concomitant effect the economy nation.

The model has the following limitations:

(i) depends on unrealistic assumptions.

(ii) no time specification of attack between the predator and prey population. There is no attraction to some equilibrium point; any perturbation in the model will continue to cycle at new amplitude until a new force acts on the model.

RESEARCH DESIGN

Well scheduled interviews were used by the research to gather information from respondents. The respondents interviewed included; farmers, Fulani herdsmen, cattle owners, leaders of youth groups in some selected villages in Ovia north-east local government area, and as well as people in the community. These people were selected for the study because of their involvement in the conflicts. In addition, journals, articles, news media and government publications were consulted for the study. Participants' observation also formed part of the research process. The research took the form of one to one interaction with respondents in the study area. This was to enable the researcher ascertain validity of claims made by respondents and also deepen the researchers understanding of the case under study.

DATA COLLECTION PROCEDURE

Data for the study was collected using questionnaire and properly scheduled interview method built on the economic characteristics of farmers and herdsmen, causes and effect of the conflict and how to resolve the problem. The researchers observed the socio-economic life of the people in the community under study. Based on the information gathered from respondents during the course of the interview, relevant conclusion was drawn. Most of the respondents played a huge role in giving the researchers key information they also led the researcher to the other persons who had vital information which aided the researcher in the course of this study. A few of the villagers gave documents to back up their claim with regards to the questions asked. During the course of the interview, English language and the local language were used interchangeably during the interview process. The information gathered were subjected to logistic test equations using the Lotka-Volterra Model based on a type 1 functional response which is the least realistic type of functional response that can be used in this type of problem because it does not consider any competition between prey and predator.

DEFINITION OF BASIC TERMS

Stability: The condition of being stable, and thus resistant to change, the tendency to recover from perturbation.
Ecosystem: The interconnectedness of organism (plant, animals, microbes) with each other and their environment.
Derivative: Having a value that depends on an underlying asset of variable value.
Exponential growth: The rate of change per unit of time of the value of a mathematical function is proportional to the function's current value.
Logistic growth: When growth rate decreases as the population reaches its carrying capacity.
Conflict: A clash or disagreement often violent, between two opposing groups or individuals.
Pastoralism: This is a state of being pastoral, the raising and herding of farm animals.
Model: A simplified representation used to explain the workings of a real world system or event.

Lotka-Volterra System of Non-Linear Differential Equation

The lotka-Volterra equation, also known as the predator-prey equations, are a pair of first-order non-linear differential equations, frequently used to describe the dynamics of biological system in which two species interact, one as a predator and the other as prey. The population change through time according to the pair of equations.

$$\frac{dx}{dt} = \alpha x - \beta xy \qquad (7)$$

$$\frac{dy}{dt} = \delta xy - \gamma y \qquad (8)$$

x = number of prey, y= number of predator and t= time.

$\frac{dy}{dt}$ and $\frac{dx}{dt}$ represents the instantaneous growth rates of the two populations where $\alpha, \beta, \gamma, \delta$ are positive real parameters describing the interaction of the two species. The Lotka-Volterra systems of equations are an example of a Kolmogorov model, which is a more general framework that can model the dynamics of ecological systems with predator-prey interaction, competition, disease and mutualism.

General assumptions of the prey- predator model

This model presents some assumptions as to how predator and prey populations operate, interact and evolve or disappear.

The prey population has always more than required food (the grasses which feed on soil unlimitedly).

Food supply for predator depends on how much size the prey population has.

The rate of change of population remains proportional to its size, in this process environment becomes unhelpful for any species.

The predator has unlimited appetite and this process is cyclical, the generation of both species continue to interact this way (cooke et al, 1981).

MODEL FORMULATION

We now present the model description, formulation and analysis. Consider a prey - predator model in which the herdsmen are the predator specie while the farmers are the prey species. Let $Y_1(t)$, $Y_2(t)$ and $P(t)$ represent the population of the farmers, herdsmen and cattle at any time *t*. The main feature of the model is that two different functional responses of the predator are incorporated in the model to represent the difference in the way the predator feeds on each of the prey species. Terms representing logistic growth of the prey species in absence of the predator are included in the prey equations. Inter-specific competition among the prey species is also included in the model. The model has 3 nonlinear autonomous ordinary differential equations describing how the population densities of the 3 species would vary with time.

Basic Assumptions, Variables and Parameters of the model

(a) The assumptions

The following assumptions are made in order to construct the model:

(i) The species live in an ecosystem where external factors such as droughts, fires, epidemics are stable or have a similar effect on the interacting species.

(ii) One prey is easy-to-capture by the predator, while the other prey has adopted anti-predator behaviour and so it requires a lot of searching and handling time for the predator to capture it.

(iii) The rate of herdsmen poaching of the farmers is on average constant per unit time and so it is represented as constant effort harvesting of the prey.

(iv) There is logistic growth of the prey in absence of the predator. That is the population of the prey would increase (or decrease) exponentially.

(v) The rate of increase of the predator population depends on the amount of prey biomass it converts as food.

(b) The Variables

The following variables are used in the model:

(i) $Y_1(\mathrm{t})$ - the population of the farmers at time *t*.

(ii) $Y_2(\mathrm{t})$ - the population of the cattle and crop at time *t*

(iii) $P(\mathrm{t})$ -. the population of the herdsmen at time *t*.

For simplicity, let Y_1, Y_2 and P be represented as $Y_1(\mathrm{t})$, $Y_2(\mathrm{t})$ and $P(\mathrm{t})$.

(c) The parameters

The following are the parameters used in the model:

r_1 *and* r_2 are per capita intrinsic growth rates for prey Y_1 and Y_2 respectively.

(ii) k_1 and k_2 are carrying capacities for prey Y_1 and Y_2 respectively.

(iii) α_1 *and* α_2 are coefficients for inter-specific competition between prey Y_1 and Y_2 respectively.

(iv) a_1 and *c* are capturing rates of predator P on Y_1 and Y_2 respectively.

(v) $\frac{b_1}{a_1}$ and $\frac{d_1}{c}$ are the predator's handling time on prey Y_1 and Y_2 respectively.

(vi) *E* is constant effort harvesting rate of prey Y_1.

(vii) *e* is natural mortality rate Of predator P.

(viii) d_2 measures the effect of anti-predator behaviour of prey Y_2.

(ix) λ_1 and λ_2 are coefficients which measure the predator's efficiency to convert prey biomass of Y_1 and Y_2 respectively into fertility (reproductively).

DERIVATION OF PREDATOR-PREY MODEL

Predator-prey models are used by scientist to predict or explain trends in the human species and animal population. The goal of this seminar is to explore the behavior of a simple model to explain the conflict between herders and farmers. Consider a population of farmers and cattle herders.

Suppose Y_1 refers to the number of farmers at any time t and Y_2 refers to the number of cattle herders. A simple predator-prey model assumes:

(i). Farmers have unlimited food supply, if there were no herders their number $y_1(t)$ would grow exponentially giving rise to the differential equation $y' = ay_1$. (9)

(ii) Actually y_1 is decreased because the farmers were being killed by the herders; say at a rate proportional to $-y_1y_2$, where y_2 is the number of herders. Hence,

$$y_1' = ay_1 \text{ -}by_1y_2 \ \ where \ \ a > 0, \ b > 0. \tag{10}$$

(i) If there were no farmers then $y_2(t)$ would exponentially decrease to zero, $y_2' = ly_2$ (11)

(ii) However, y_2 increases by a rate proportional to the number of encounters predator had with the prey. Therefore, if we put the equation together, we have,

$$y_2' = (-l\,y_2 + ky_1y_2) \ where \ k > 0 \ and \ l > 0. \tag{12}$$

Putting these equations (10) and (12) in the form of Lotka-Volterra system we have

$$y_1' = ay_1y_2 - by_1y_2$$
$$y_2' = ky_1y_2 - ly_2$$

(13)

Where the critical points are the solution of

$y_1(a-by_2) = 0,$ (14a)

$y_2\,(ky_1-l) = 0$ (14b)

are given by $(y_1, y_2) = (0,0)$ and (lk, ab). respectively

At (0,0) the linearized system, is $y' = ay_1, \ y_2' = (-ly_2)$. (14c)

The eigenvalues are $\lambda_1 = a>0$ and $\lambda 2 = -l<0$ which will give rise to a saddle point (position of authority or control) at the critical point (lk, ab).

we set; $y_1 = lk + \tilde{y}_1$

$y_2 = ab + \tilde{y}_2$ (15)

$$y_1' = (y_1 + \text{lb})(\text{a} - \text{b}(\tilde{y}_2 + \text{ab})) \tag{16}$$

$$y_1' = (y_1 + \text{lb})(-b\tilde{y}_2)$$

$$y_2' = (y_2 + \text{ab})(\text{k}(y_1 + \text{lb}) - \text{l}) = (\tilde{y}_2 + ab)k\tilde{y}_1 \tag{17}$$

dropping the two non-linear terms we obtain the linearized system

(a) $y_1' = -lbk\,\tilde{y}_2$

(b) $y_2' = akb\,y_1$

In matrix form we see that (a) and (b) will be

$$y' = \begin{bmatrix} 0 & -lbk \\ akb & 0 \end{bmatrix} y \tag{18}$$

which by characteristic polynomial equation gives,

$|A - \lambda I| = 0$ (19)

Where,

$$\begin{vmatrix} -\lambda & -lbk \\ abk & -\lambda \end{vmatrix} = 0 \tag{20}$$

which has a centre at the origin and at infinity because

$\lambda = \pm \sqrt{l}ai$, are imaginary eigenvalues.

by simply multiplying the left hand side of (a) by the right hand side of (b) we have

(c) $akb\ \tilde{y}_{12} + lbk\ \tilde{y}_{22} = \text{constant}$

From which we obtain the ellipses.

Hence, this critical point of the linearized system is a center

(See Figure 1).

It can be shown by a rather complicated analysis that the non-linear system (7) and (8) also has a center at (yk, ab), surrounded by closed trajectories (not ellipses), not a spiral point.

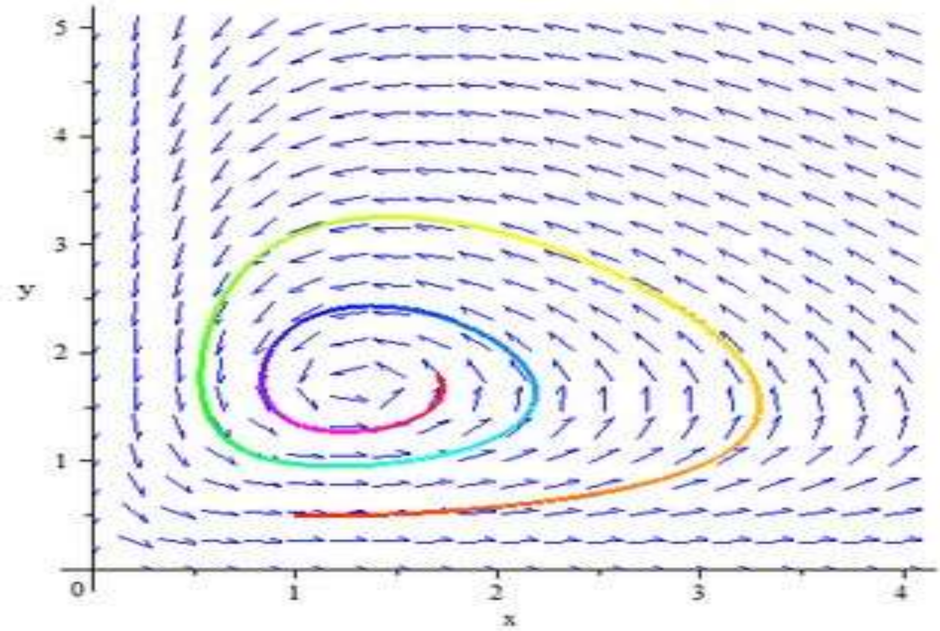

Figure 1. Ecological equilibrium point

(Trajectory of the linearized lotka-volterra system (18))

The model equations for herdsmen and farmers conflict

From the model description, assumptions and definition of variables and parameters as stated above, the equations that represent the dynamics of the herdsmen and farmers are formulated as below,

$$\frac{dy_1}{dt} = r_1 y_1\left(1-\frac{y_1}{k_1}\right) - \alpha_1 y_1 - \left(\frac{a_1 y_1}{1+b_1 y_1}\right)p - E \qquad (21)$$

$$\frac{dy_2}{dt} = r_2 y_2\left(1-\frac{y_2}{k_2}\right) - \alpha_2 y_1 y_2 - \left(\frac{c y_2}{1+d_1 y_2 + d_2 p}\right) \qquad (22)$$

$$\frac{dp}{dt} = ep + \lambda_1\left(\frac{\alpha_1 y_1}{1+b_1 y_1}\right)p + \lambda_2\left(\frac{c y_2}{1+d_1 y_2 + d_2 p}\right)P \qquad (23)$$

where all parameters in the model are positive.

For ease of computations, non-dimensionalization of the model represented by equations (21), (22) and (23) is carried out to reduce the number of parameters by setting

$X_1 = By_1, \quad X_2 = d_2 y_2 \; and \; X_3 = d_2 p$ so that equation (21) – (23) become

$$\frac{X_1}{dt} = r_1 X_1\left[\left(1-\frac{X_1}{b_1 K_1}\right) - \frac{\alpha_1 X_1}{r_1 d_1} - \frac{a_1 X3}{r_1 d_2(1+X_1)} - {}^{E}/_{r_1}\right] \qquad (24)$$

$$\frac{X_2}{dt} = r_2 X_2\left[\left(1-\frac{X_2}{d_1 K_2}\right) - \frac{\alpha_2 X_1}{r_2 b_1} - \frac{cY}{r_2 d_2(1+X_2+Y)}\right] \qquad (25)$$

$$\frac{dy}{dt} = eY\left[-1 + \frac{\lambda_1 a_1 X_1}{e b_1(1+X_1)} + \frac{\lambda_2 c X_2}{e d_1(1+X_2+Y)}\right]. \qquad (26)$$

Also by setting

B= ${}^{1}/_{K_1 b_1}$, $\delta_1 = {}^{\alpha_1}/_{d_1 r_1}$, $g_1 = {}^{a_1}/_{r_1 d_2}$, $M = {}^{E}/_{r_1}$,

$\beta_2 = {}^{1}/_{K_2 d_1}$, $\delta_2 = {}^{\alpha_2}/_{r_2 b_1}$, $g_2 = {}^{c}/_{r_2 d_2}$,

$h_1 = {}^{\lambda_1 a_1}/_{e b_1}$, $h_2 = {}^{\lambda_2 c}/_{e d_1}$

We have

$$\frac{dX_1}{dt} = r_1 X_1\left[(1-\beta_1 X_1) - \delta_1 X_2 - \frac{g_1 Y}{1+X_2+Y} - M\right] \qquad (27)$$

$$\frac{dX_2}{dt} = r_2 X_2 \left[(1 - \beta_2 X_2) - \delta_2 X_1 - \frac{g_2 Y}{1 + X_2 + Y} \right] \quad (28)$$

$$\frac{dY}{dt} = ey \left[-1 + \frac{h_1 X_1}{1 + X_1} + \frac{h_2 X_2}{1 + X_2 + Y} \right] \quad (29)$$

Equation (28) has 12 parameters compared to (21) with 15.

Another extremely important plot stemming from the model is the predator-prey cycle chart, representing periodic activity in the population fluctuation. This diagram is generated by plotting the y-t and x-t curves on the same plot, showing the fluctuation of predator and prey populations with respect to time on the same chart, therefore important characteristics of time can be analyzed. Figure 2 below depicts a sample of predator-prey cycle chart in odighi, Ovia local Government Area of Edo State.

In this MATHLAB display it is seen that as time progresses (in years), predator and prey populations clearly fluctuate at cyclic interval. Notice that as the prey population peaks, predator population begins to rise rapidly, yet as the predator population rises, the prey population falls rapidly.

Then follows a longer period as the prey population must slowly repopulate and the predator population falls drastically. This cycle repeats itself over and over in reality, which is why biological Mathematicians can attempt to recreate the pattern mathematically. By setting $y_1 = N_1$ and $y_2 = N_2$ we illustrate the cyclic representation of the predator prey in figure 2 below.

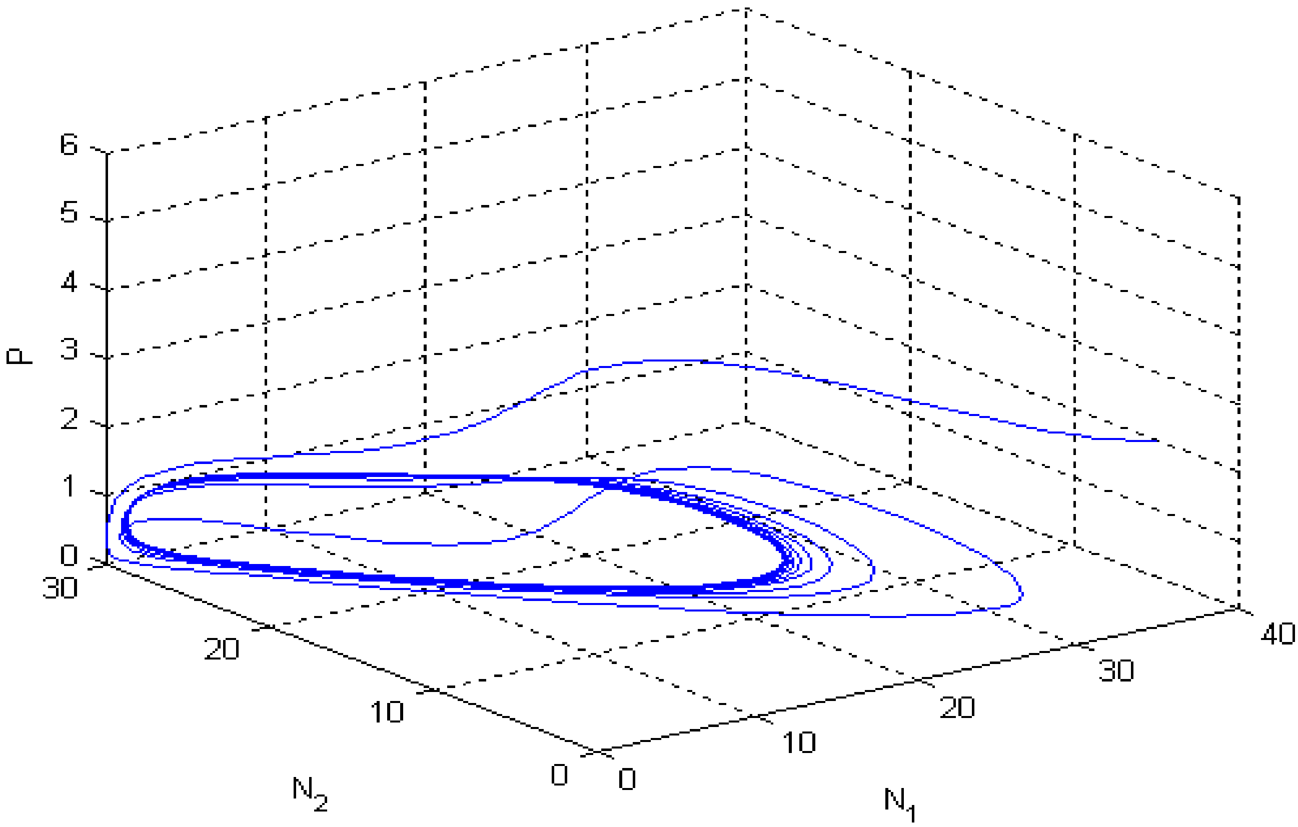

Figure 2: Predator-Prey Cycle Chart

RESTRICTIONS AND LIMITATIONS

Although the Lotka-Volterra model is the best model currently available to accurately present mathematically the population variation in the dynamics of a predation relationship, there is the need to explore other types of models. All of the assumptions that go into this kind of model development naturally inflict some 'holes'. At the same time restrictions and limitations exist in the assumptions model construction.

In my view, the greatest limitation this model has maybe its lack of relevance to the other of predation relationships.

CONCLUSIONS AND FUTURE WORK

This seminar if nothing else has showed that human problems can always be solved mathematically by first modeling them into Differential Equations. Therefore, one of the main conclusions of this research is that in the realm of biological mathematics, it is possible to mathematically represent the population variations of a predation relationship to a certain extent of accuracy. This can be done using the Lotka-Volterra Model. By transforming the problem into system of linear first order differential equations we can represent the real life problems analytically and graphically when there are cyclic fluctuations in population species.

There is a level of error in theoretical versus numerical computation of solution, but the overall form, structure, and time of the fluctuations should be consistent. Note however, that errors are most probably due to external variables unaccounted for in the formulation of the model.

After analyzing the data collected, it was found that Herdsmen and farmers populations fluctuate, due to the periodic oscillations. The information provided in this model can be very useful to life Scientists that focus on extinction prevention, or co-evolution studies. Apart from being able to determine population fluctuation characteristics, the model can provide extinction prevention strategies. Where extinction is naturally possible, preparing them ahead of time to do intense tagging and developing natural habitats for the cattle may facilitates safety precautions.

Co-evolution researchers can also use this model to determine the number of cycles required for each species to develop new means of survival that can adapt them to competition. There are infinitely many new and diverse models that can be developed from this very basic starting point given here today. The Lotka - Volterra Model can serve as a stepping stone into the field of biological and ecological world mathematically.

Other options for future work include studying other predation relationships for which the Lotka-Volterra Model may apply, particularly for species which rely on one another for resources while external variables are kept constant.

This model has simply broken the ground of endless possibility in the world of biological mathematics which is the backbone of applying science to solving real life problems.

The graph graphs show that prey N_1 and predator P become extinct if the harvesting rate E is greater or equal to its intrinsic growth rate r_1

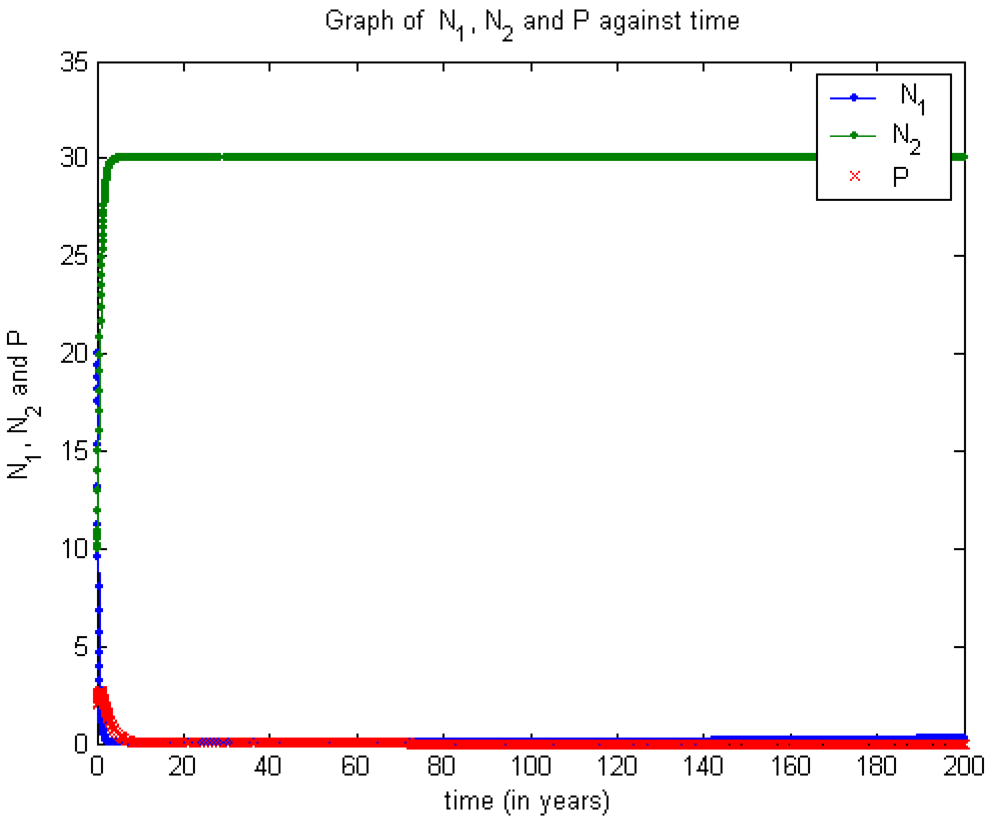

Figure 3: Graph of prey N_1, N_2 and predator P against time when $r_1 = E = 3$ and $r_2 = 2$.

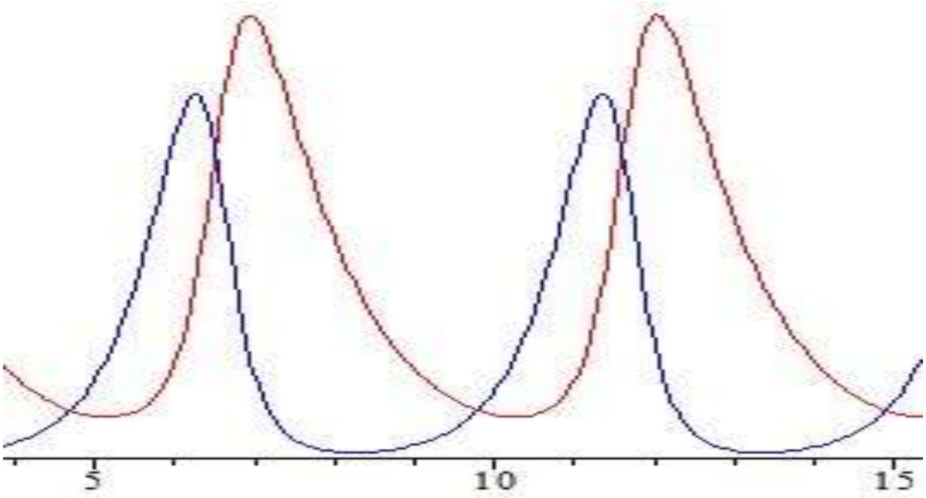

Figure 4: Graph of $N_1, against\ P$

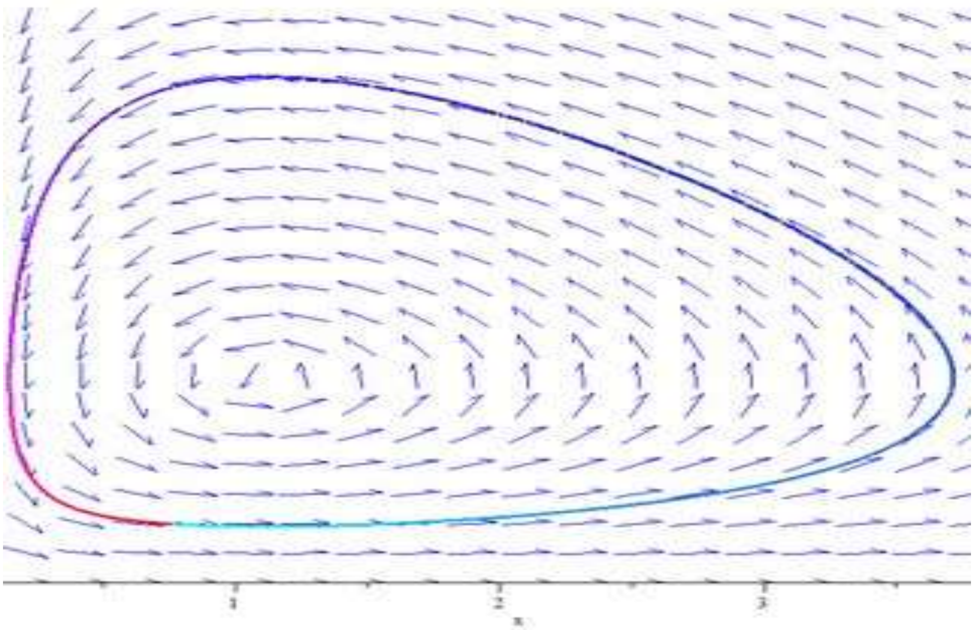

Figure5: Graph of $N_2\ against\ P$

ACKNOWLEDGEMENT

I remain grateful to the Almighty God for making me who I am and for giving me this opportunity to present this seminar. I thank my amiable Vice Chancellor, Prof. I. O. Onimawo for the unparallel transformation he has started in this University and for daring to create a new academic frontal that challenge both the students and lecturers to see the need for technological approach to teaching and learning which is gradually gaining grounds.

My sincere thanks go to my lovely Dean, Prof. I. O. Asia for making it possible for me to present this seminar. Also worthy of my thanks are the chairman and members of the Faculty seminar committee for the excellent job they are doing to ensure that the exercise continues in spite all odds. My thanks go to Prof. U. S. U. Aashikpelokhai my mentor, Prof F. M. Okoro, Prof. C. E. Abhulimen, Dr. A. O. Elakhe, my current Head of Department, Dr. A. O. Isere, Dr. J. T. Erimafa, all the other lecturers, the Departmental Secretary other non-academic staff and students of Mathematics Department who are so dear to me.

I am completely indebted to my lovely wife and children who are always there for me no matter the situation. I remember with nostalgia, my late parents Mr. A. M. Okooboh and Mrs. Grace Agbeboh of the blessed memory. My friend and well-wishers are not left out in my gratitude. I wish you all the best in your life Endeavour.

To God be the Glory.

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