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# Analyzing Common Errors in Differentiating and Integrating Functions as Basis for Supplemental Calculus Learning Tool

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**ABSTRACT:** Calculus is one of the most interesting fields of Mathematics yet difficult to pass. In this study, the common errors committed by the students were analyzed and categorized based on Newman's error analysis guide. Using the quantitative research method, 10 Bachelor of Secondary Education major in Mathematics students enrolled in Calculus I and II were purposively chosen. The study hypothesized that no significant effects of the errors committed when classified according to types of errors and math fields to the overall performances in Calculus I and II of the students. Data collected were analyzed and interpreted using Mean Percentage Scores and Structural Equation Modelling. Based on the results, students commit errors mostly in differentiating functions involving logarithms. In terms of integration, students commit errors more on algebraic functions. Based on Newman's error analysis, errors committed by mathematics major students are categorized into comprehension in both differentiation and integration processes. The result reveals that while students are equipped with the necessary skills needed for the transformation, process, and encoding, comprehension of the pre-requisite concepts should be strengthened and given attention specifically in algebraic and logarithmic functions. Finally, an instructional material addressing the identified common errors was developed.

**KEYWORDS:** Calculus, Common Errors, Differentiation, Integration

## INTRODUCTION

Calculus is one of the most interesting fields of Mathematics. It requires competencies such as Algebra, Trigonometry, and other mathematical skills. Its application is evident in engineering, architecture, health, machine learning, economics, and other related fields. There are two branches of Calculus that involve the study of differentiation particularly the rate of change and the integration part which measures the area under the curve and volume of an object. In the basic education curriculum in the Philippines, the study of calculus is included in the academic track in the Senior High School program, particularly in Science, Technology, Engineering, and Mathematics (STEM) strand.

Since the subject is only included in some select strands, many students in the tertiary level (higher education) perceived this subject as most difficult to pass. According to Ahmad (2017), 30% of the students in every semester failed this subject due to poor study habits and negative attitudes toward learning (Casinillo, 2019). Similarly, Kauffman (2011) identified several factors affecting the poor performance of the students in this subject. Factors contributing to poor performance include understaffing, inadequate teaching/ learning materials, lack of motivation and poor attitudes by both teachers and students, and retrogressive practices. Improving on these factors and sensitization of the local community to discard practices that prohibit students' effective participation in learning mathematics could improve performance in Mathematics (Mbugua, 2012).

Several studies have been conducted to determine the factors affecting the performance of students in calculus (Muzangwa & Chifamba, 2012; Yerizon, 2019) but little is known about the way students solve the problem and the common mistakes committed by them which would serve as input for an intervention program or project. Hence, this study will focus on analyzing the errors in solving problems among students enrolled in Calculus 1 and 2 and classify these errors using Newman's (1977, 1983) indicators.

Error analysis is an analysis that finds the pattern of mistakes committed by the students specifically in solving mathematical equations or problems and relating it to some probable causes (Herholdt & Sapire, 2014).

## OBJECTIVES OF THE STUDY

This study was conducted to analyze the errors committed by students at Laguna State Polytechnic University enrolled in Calculus 1 and 2. In particular, it will determine the following: the common errors in committed by the students in solving differentiation and integration problems; classify the errors based on Newman's Indicators; determine the significant effect of errors to the Calculusperformance of the students; determine the significant contribution of competencies in Algebra, Geometry and

Trigonometry on the committed errors of the students; and develop a supplemental tool for minimizing the errors committed by the students.

## MATERIALS AND METHODS

The study used the quantitative research method specifically the descriptive-evaluative design. Descriptive-evaluative research focuses on specifying the present condition of the behavior without providing any intervention. It likewise points out the need for providing a description on the current status and deeply explicating the characteristics of the behavior (Siedlecki, 2020) . Similarly, evaluative process is used in order to explain the commonly committed errors by the students in differentiating and integrating functions. This study used the processes done by Brown (2006), Isik & Kar (2012) and Muzangwa & Chifamba (2012) about error analysis as well as Triliana and Asih (2019), and Muntazhimah, M., Prabawanto, S., & Turmudi, T. (2023). Two sets of instruments will be used in this study. One test for measuring competencies in differentiating functions while the other one is the test measuring the competencies in integrating functions. Both tests include four constructs such as competencies in differentiating and integrating functions in terms of the following concepts: algebraic, logarithmic, trigonometric, and exponentials. For each exam, the answers of the students will be analyzed and errors will be categorized using Newman's indicators such as *comprehension, transformation, process skills*, and *encoding*. For each assessment, 10 student respondents who expressed willingness to participate in the study were given 1.5 hours to accomplish the questions.

The questions are focused only on some abstract expressions and not on problem-solving to extract their basic understanding of the concepts and the applications of these concepts in the next higher level of thinking. Students-respondents are informed that they are given opportunities to withdraw in participating in the study if they feel so. Likewise, they are informed that all the information provided by them would be treated with the utmost confidentiality. More so, they are being informed that the data gathered would be solely used for the research purpose only and data will not be treated singly but collectively. Data collected were analyzed and interpreted using some descriptives and inferential statistics which include mean, median, mode, standard deviation, skewness, kurtosis, and percentages. Inferential statistical treatments include the Chi-square test of independence, Analysis of Variance (Anova), and Regression analysis. After the analysis, instructional material that serves as supplementary material is crafted focusing on combating the common errors committed by the students. The supplementary material is also converted to an electronic file which was given also to the respondents after the study.

## RESULTS AND DISCUSSION

Table 1 shows the commonly committed errors by the students in differentiating and integrating functions. Based on the table, common errors committed by the students involved the basic concepts of the pre-requisites such as basic grouping symbols, integers, and application of rules of exponents and logarithms.

**Table 1. Common Errors Committed by the Students**

| **Commonly Committed Errors** |
|---|
| Constant of Integration (+C) |
| Differentiating negative exponents |
| Forgot to apply the parenthesis in the -sin function |
| Forgot to differentiate the terms. |
| Forgot to distribute the exponent in the denominator |
| Forgot to include cosine r as a function of cosine |
| Forgot to include parenthesis |
| forgot to include the exponent in the final answer |
| Integrating natural logarithms |
| Integrating x squared after factoring the arctan function |
| Interchanging the terms but forgot to differentiate the other factor. |
| Laws of logarithms |
| Missing the negative sign before the numerical coefficient |
| Multiplying indices to the numerical coefficient |
| Parenthesis in the final answer |
| Placing parenthesis |
| Putting a negative on the final answer |
| Putting a parenthesis in the solution |
| Putting a variable x to the final answer. |
| Rule of Negative numbers |
| Rules of Exponents |
| Sign of the final answer has been changed. |

| |
|---|
| The signs of 2nd and 3rd terms interchanged. |
| Typing the variable in the exponent |

Based on the table, 55% of the respondents commit errors when it comes to differentiating functions involving logarithms followed by differentiating functions involving exponentials (50%), trigonometric (40%), and algebraic (35%). As to integration, algebra-related indicators ranked first (35%) when it comes to committed errors by the students. This is followed by logarithmic, exponential, and trigonometric functions. Results also show that a large number of errors have been committed to differentiation as compared to integration. Also, the same percentage of errors have been committed by the students in both differentiating and integrating algebraic functions.

**Table 2. Common Errors Committed by the Students Based on Math Fields**

| **Math Fields** | Common Errors | |
|---|---|---|
| | **Differentiation (%)** | **Integration (%)** |
| Algebra | 35.00 | 35.00 |
| Logarithmic | 55.00 | 20.00 |
| Exponential | 50.00 | 15.00 |
| Trigonometric | 40.00 | 15.00 |
| Average Percentage | 45.00 | 21.25 |

Considering the type of errors committed, a large portion of the errors committed is related to comprehension in both differentiation and integration by the students. Meanwhile, students do not show errors when it comes to transformation. Comprehension errors or errors committed due to misunderstanding of the concepts or principles happened when a student grasps and knows what is needed to be solved but actually missed some or many components of the concept, eventually, the student cannot proceed to solve the problem correctly. In addition, according to Fitriani, et al. (2018) process skill error is an error of students in picking appropriate rules or procedure. It also refers to errors committed in the computation process. The result of the study is in conformity with Pomalato, et al. (2020) found that understanding and process skills errors are the two committed errors in solving mathematical problems specifically in Calculus.

**Table 3. Common Errors Committed by the Students Based on Classification**

| **Error Classification** | **Common Errors** | |
|---|---|---|
| | **Differentiation (%)** | **Integration (%)** |
| Comprehension | 27.5 | 15 |
| Transformation | 0 | 0 |
| Process Skills | 10 | 5 |
| Encoding | 7.5 | 1.25 |

One of the students' errors is presented in the figure 1. In the figure, the student forgot to multiply the indices to the numerical coefficient of the given term in the given algebraic function. Supposedly, student should apply the derivative rule $f'x = nx^{n-1}$ and that the solution is supposedly $f'x = 5x^{-2} + 4x^{-3} - 3x^{-4}$ resulting to $f'x = -10x^{-3} - 12x^{-4} + 12x^{-5}$ or $f'x = -\frac{10}{x^3} - \frac{12}{x^4} + \frac{12}{x^5}$. However, the student mistakenly apply the derivative rule to the denominator only and did not consider the numerical coefficient and the rules of exponents. In Figure 2, the proper use of parenthesis is neglected making the entire solution and final answer incorrect. In Figure 3, the student mistakenly placed a negative sign on the final answer which makes answer incorrect. However given the emphasis on the solutions, the student followed properly the process of solving the problem except committing encoding mistake in the final answer.

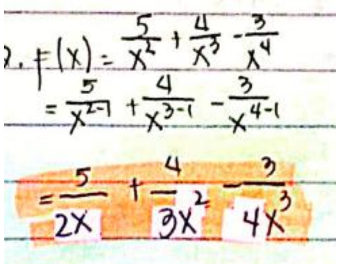

**Figure 1.** Example of Student's Comprehension Error in Differentiating Functions

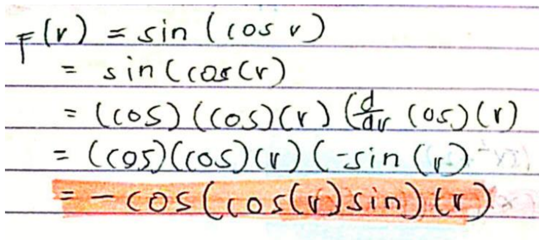

**Figure 2.** Example of Student's Process Skill Error in Differentiating Functions

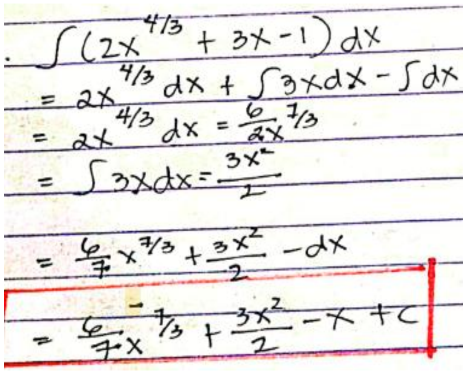

**Figure 3.** Example of Student's Encoding Error in Differentiating Functions

The performances of students in Calculus I and Calculus II obtained the mean values of 87.351 and 89.3 respectively both interpreted as Satisfactory. The skewness value of -0.77 and -0.47 fall on the category of normal distribution. The result shows that there is a consistent performance of the students in Calculus I and Calculus II.

**Table 4. Performances of Students in Calculus I and Calculus II**

| **Statistical Measures** | ***Calculus 1*** | **Calculus 2** |
|---|---|---|
| Mean | 87.351 | 89.3 |
| Standard Deviation | 5.79 | 4.72 |
| Sample Variance | 33.47 | 22.23 |
| Kurtosis | -0.37 | -1.73 |
| Skewness | -0.77 | -0.47 |

The outer loading for errors committed when classified according to Newman's error analysis fall ranges from 0.65 to 0.78 which are categorized as acceptable and within the threshold (Afthanorhan, 2013). Based on the result of the path coefficient (-0.798), the errors committed by the students in differentiating functions when grouped according to Newman's error analysis classification have shown negative effects on their overall performance in Calculus I by 63.6% where errors categorized to Process Skills show a large contribution. Likewise, the results of the path analysis revealed that the negative effect is significant in the three types of errors.

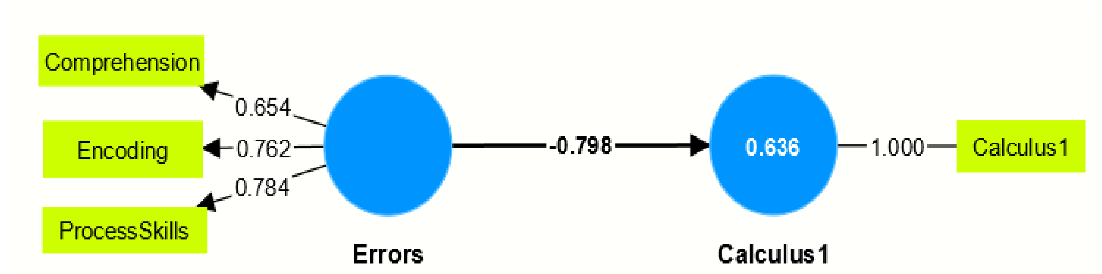


**Figure 4.** Structural Model showing Path Coefficient, Loadings on the Errors Committed by the students in Differentiating Functions, and their Overall Calculus 1 Performance

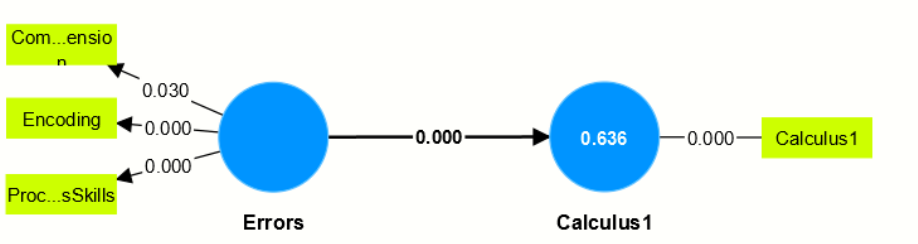


**Figure 5.** Structural Model showing the p-values on the Errors Committed by the students in Differentiating Functions and their Overall Calculus 1 Performance

The path model shows the interplay of the exogenous and endogenous variables for the effects of errors committed to differentiating functions classified as algebraic, exponential, logarithmic, and trigonometric. Based on the figures, the overall path coefficient from errors per field to the Calculus 1 performance of the students is -0.862 indicating the negative effects of the errors per field on the overall Calculus 1 performances of the students. The result also reveals that among the outer loadings, errors committed to differentiating trigonometric problems expressed the greatest correlation value to the construct while errors in differentiating algebraic problems have a moderate correlation to the construct. As revealed by the p-values which are all less than the threshold of 0.05, all errors committed by the students are significantly affecting the overall performance of the students in Calculus I.

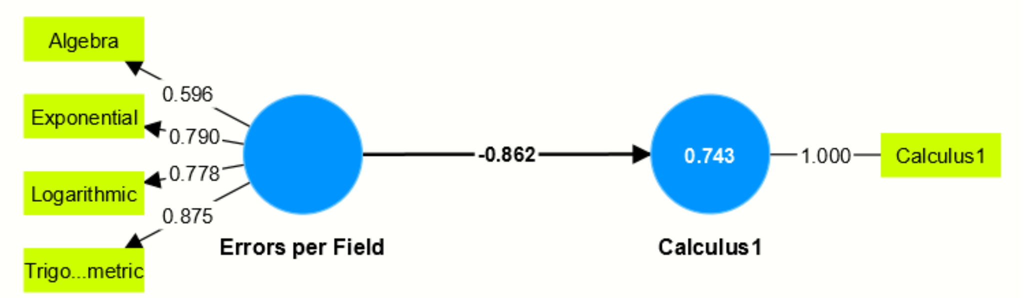


**Figure 6.** Structural Model showing Path Coefficient, Loadings on the Errors Committed by the students in Differentiating Functions as to Math Fields, and their Overall Calculus 1 Performance

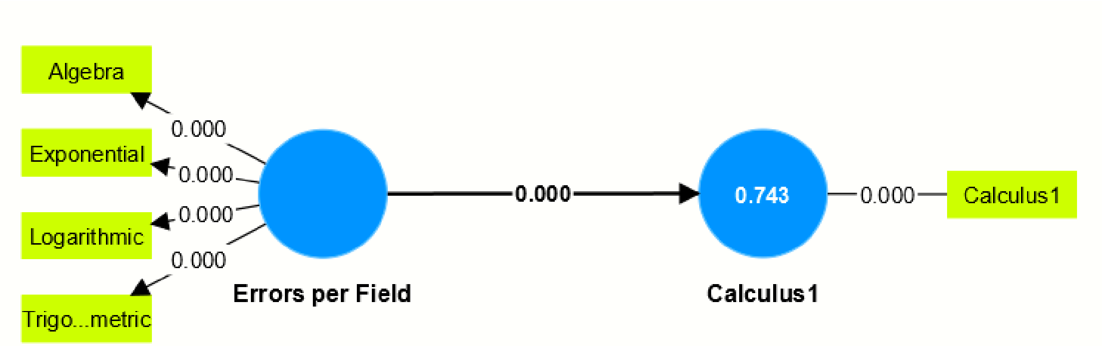


**Figure 7.** Structural Model showing the p-values on the Errors Committed by the students in Differentiating Functions as to Math Fields, and their Overall Calculus 1 Performance

The effects of the errors committed by the students and their performances in Calculus II are presented in the figures. Results show that except for errors committed in integrating trigonometric and exponential functions, the other types of errors committed by the students (Algebraic and Logarithmic) are negatively affecting the overall performance of the students in Calculus II (Integral Calculus). Based also on the p-values, the effects of errors committed to integrating algebraic and logarithmic functions are significant.

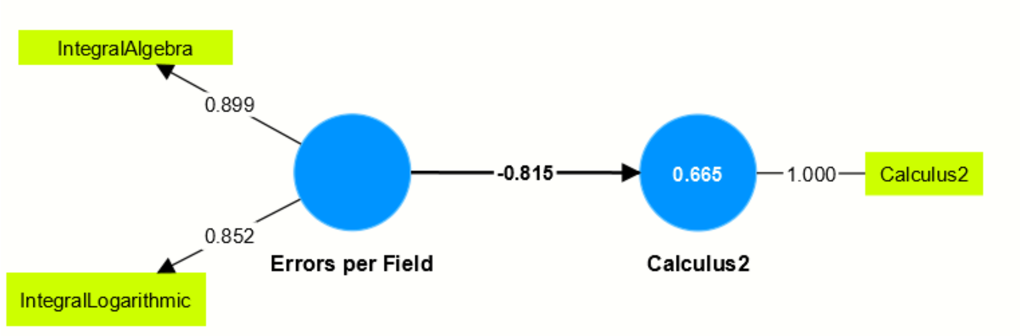


**Figure 8.** Structural Model showing Path Coefficient, Loadings on the Errors Committed by the students in Differentiating Functions, and their Overall Calculus 2 Performance

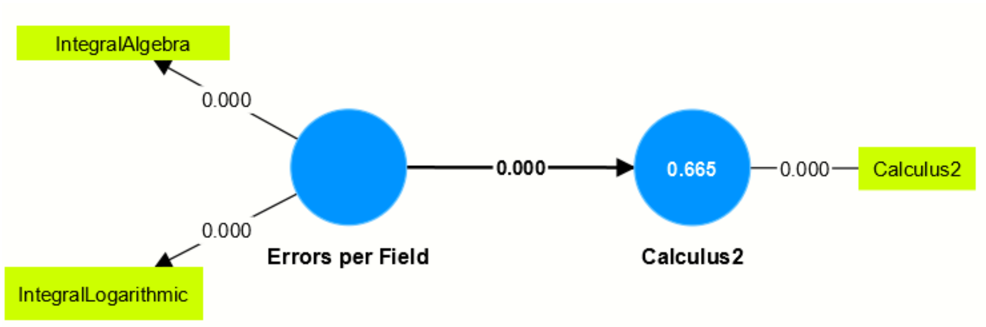


**Figure 9.** Structural Model showing the p-values on the Errors Committed by the students in Differentiating Functions and their Overall Calculus 2 Performance

Using Newman's classification of errors, the results presented in the figures show that comprehension and process skills errors committed by the students negatively affect their overall Calculus II performances. The p-values of <0.05 justify the significance of the results.

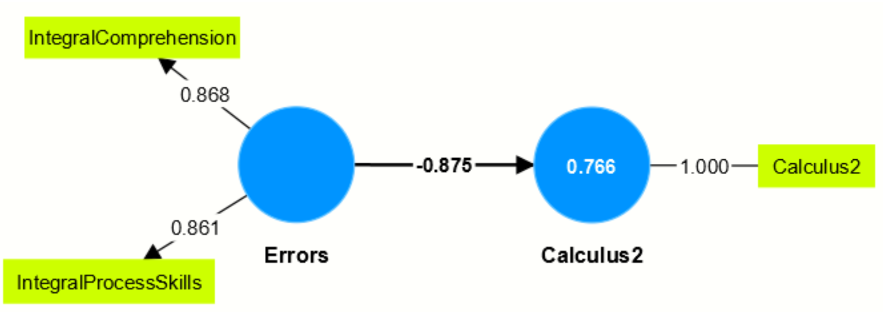


**Figure 10.** Structural Model showing Path Coefficient, Loadings on the Errors Committed by the students in Differentiating Functions as to Math Fields, and their Overall Calculus 2 Performance

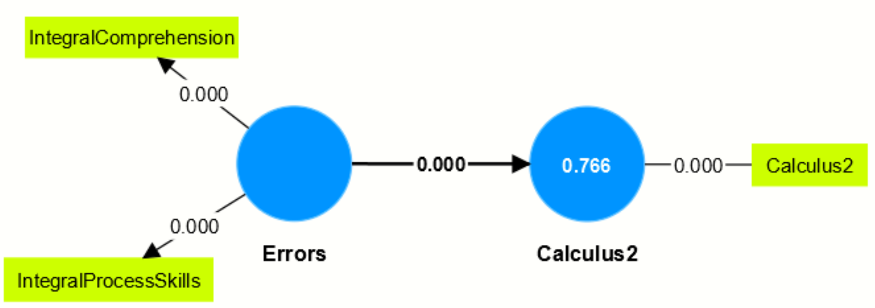


**Figure 11.** Structural Model showing the p-values on the Errors Committed by the students in Differentiating Functions as to Math Fields, and their Overall Calculus 2 Performance

The relationship between the overall performances of the students in Calculus I and Calculus II is found significant and the performance of students in Calculus I affects their performance in Calculus II which focused more on the integration and its applications to Geometry, particularly areas, and volumes.

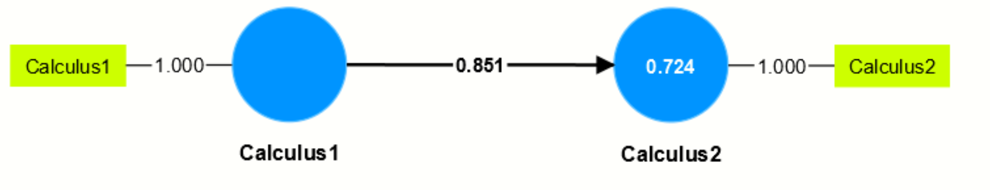


**Figure 12**

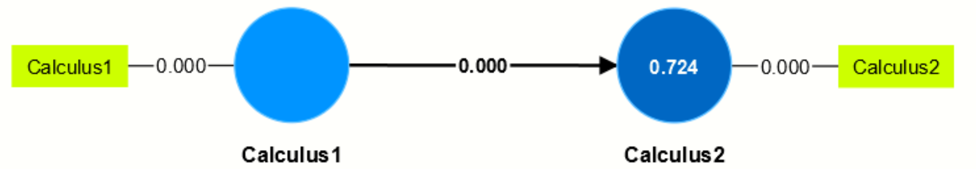


**Figure 13.** Relationship between the Calculus 1 and Calculus 2 Performances of the Students.

**Supplemental Learning**

Based on the common errors identified in the study, supplemental material is crafted. The supplemental material consists of different activities for the differentiation and integration of functions. The material consisted of 14 sets of activities focused on basic applications of derivative rules and integration rules. Differentiation topics include rules for algebraic, exponentials, logarithmic, trigonometric, chain rule, product rule, quotient rule, and implicit differentiation. Activities for integration focused on basic algebraic, exponentials, trigonometric, logarithmic, integration by parts, integration by substitution, integration by partial fractions, and definite integration, finding the area and volume of geometric objects.

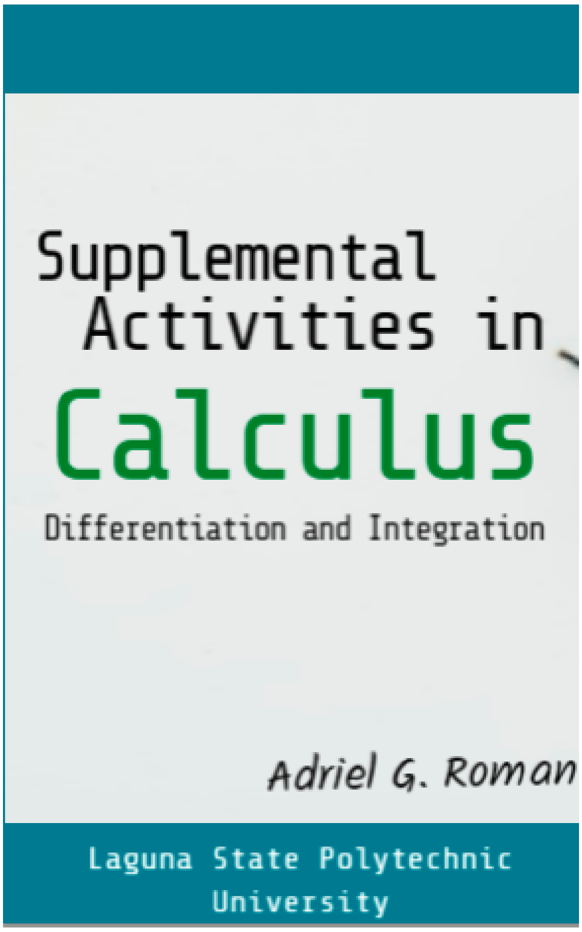


**Figure 14.** Supplemental Material Developed

## CONCLUSION AND RECOMMENDATIONS

Errors committed by the students in differentiating and integrating functions came from the basic foundation and not from the advanced concepts of Calculus. These errors classified as comprehension, process skill, and encoding negatively affect the overall performance of students in Calculus I and II. For these reasons, the basic skills of the students particularly in Algebra, Trigonometry, Logarithms, and Exponents should be strengthened as they affect significantly the performances of students in Calculus.

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